Problem Set IV: Due March 4, in class

1.) a.) Consider a chunk of collisionless, self-gravitating matter in one dimension. Here, take a "chunk" to be:

$$f = \begin{cases} f_0, & u_0 - \Delta \mathrm{v} < \mathrm{v} < u_0 + \Delta \mathrm{v} \\ 0, & \text{elsewhere} \end{cases}.$$

Here, f_0 is constant. Take $u_0, \Delta\mathrm{v}$ fixed. Using the Vlasov-Poisson system, calculate the marginal stability criterion for Jeans instability. Compare your result to the case discussed in class for a self-gravitating gas.

b.) Now consider a plasma, with

$$f = \begin{cases} f_{\max} + f_0, & u_0 - \Delta \mathrm{v} < \mathrm{v} < u_0 + \Delta \mathrm{v} \\ f_{\max}, & \text{elsewhere} \end{cases}.$$

Consider $f_0 > 0$ and $f_0 < 0$. $f_{\max}$ is the usual Maxwellian. Of course $f_{\max} + f_0 > 0$, for all v. What is the marginality condition now? Relate your result to the bunching condition discussed in class for the beam-plasma interaction. Hint: Consider the sign of the dielectric function.

c.) For collisionless, self-gravitating matter with an initially Jeans unstable distribution, discuss how the instability might saturate. Hint: Consider simple quasi-linear analysis.

2.) a.) Derive the *resonant* quasilinear diffusion equation using a Fokker-Planck calculation.

b.) Explain carefully why the non-resonant diffusion effect *cannot* be derived using a Fokker-Planck approach.

3.) a.) Calculate the electron and ion heating predicted by quasi-linear theory for a spectrum of unstable ion-acoustic waves. Which is larger?

b.) How does the electron heating rate compare to the seat-of-the-pants estimate ηJ^2, using the *anomalous* resistivity (from quasi-lineary theory) for η.

4.) a.) Derive the Fokker-Planck equation for the motion of a Brownian particle in a spectrum of thermal fluctuation - induced forces, with white noise. Calculate all parameters explicitly.

b.) What pdf is the stationary state solution of this equation? What does this imply about the time-asymptotic particle distribution?

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6.) Consider an ensemble of Brownian particles distributed in a jar of water at temperature *T*. Assume the particles fall under gravity, and experience a Stokes drag.

a.) Assuming the particles quickly reach terminal velocity, derive the Fokker-Planck equation for the density of particles.

b.) Calculate the steady state density profile and show that your result agrees with that from equilibrium statistical mechanics.

7.) Consider a linearly stable current driven ion acoustic system (i.e. 1D, electrons with shifted Maxwellian at mean velocity v_0 and temperature T_e; ions with unshifted Maxwellian and temperature T_i).

a.) What condition is sufficient to *guarantee* stability?

b.) Derive the effective resistivity (i.e. electron-ion momentum transfer) using the Lenard-Balescu equation. Justify your result. How does this behave approaching marginality?

c.) Discuss how the dimensionless parameters m_e/m_i, T_e/T_i, v_0/c_s influence this resistivity.